
A.A. Povalyaev

doctor of engineering science

Joint Stock Company "Russian Space Systems", Russia

e-mail: povalyaev_aa@spacecorp.ru

Abstract. Modern educational material and scientific literature dedicated to satellite navigation describes the operating principles of ground positioning systems and satellite radio navigation systems using such terms as pseudorange and pseudodelay. Pseudorange is defined as the product of pseudodelay by light speed. Pseudodelay τpd is defined as τpd, the difference between reception time of the navigation signal in the receiver timescale and the signal transmission time on the satellite timescale. However, the aforementioned sources do not contain any explanation regarding the following issues: how the receiver determines the value of τpd, what the terms “timescale” and “time point on any given timescale” mean, and what the difference between the time according to the timescale and the actual physical time, mentioned in physics textbooks, is. Moreover, many sources define the pseudodelay P, either explicitly or implicitly, as a time interval without explaining whether it is meant to be an interval of the actual time or the time interval within any particular timescale.

Among the above-mentioned systems, nowadays the most complicated and, at the same time, the most advanced ones are the global navigation satellite systems (GNSS). Based on the critical review, the contradictions have been revealed in the paradigm used in modern educational material and scientific literature, which focus on the operating principles of the GNSS. A new paradigm based on defining the concepts of the timescale and satellite clock time is introduced. This new paradigm eliminates the revealed contradictions. A substantial simplification of the system development of the ground positioning systems is suggested based on the newly reintroduced concepts and paradigms.

Keywords: GNSS, pseudorange, pseudodelay, timescale, satellite clock time
Introduction

Nowadays several types of radio navigation systems (RNS), that are related by their conceptual fundamentals, such as global navigation satellite systems (GLONASS, GPS [1-14]), ground-based very low frequency systems (Omega, Alpha, Marshrut [4, 15]), and ground-based long wave systems (Loran-C, Chaika [4]) are operational. By their structure, all these systems are networks of either stationary or mobile radio navigation beacons (RNB) synchronously emitting navigation signals. The timestamps that are carried by these synchronically emitted signals are called the time scale of the system.

Among the above-mentioned systems, the most complicated and, at the same time, the most advanced ones are the global navigation satellite systems (GNSS). Accordingly, the description of the GNSS functioning principles requires using the most complicated conceptual fundamentals. For other RNS, the conceptual fundamentals are simpler and are considered a special case of the GNSS conceptual fundamentals.

1. Review of the concepts used in modern educational and scientific literature to describe the GNSS functioning principles

Sources [1-14] describe the functioning principles of the GNSS using the terms pseudorange and pseudodelay. Pseudorange is defined in all the sources as pseudodelay multiplied by the speed of light. Pseudodelay is defined in [1-14] as \( r_{\rho} = t_r - t_i \), the difference between time \( t_r \) of receiving a navigation signal in the timescale of the receiver and time \( t_i \) of its emission in the time scale of the navigation satellite. The method by which the receiver learns the value \( t_i \), the meaning of the terms “timescale” and “moment of time in a certain scale”, and how the timescale differs from the physical time used in physics textbook is not revealed in [1-14]. Moreover, in the works [1-9], explicitly or not, the pseudorange \( r_{\rho} = t_r - t_i \) is treated as a time interval, without explanation whether or not it is an interval of physical time or a time interval in a certain scale.

Fig. 1 is used, explicitly or not, to describe the GNSS functioning principles in [1-14]. Here, it was taken from the textbook [4]. Similar figures are used for these purposes in [2, 8, 9, 12].

![Fig. 1. Description of pseudodelay as a time interval](image)

The GNSS functioning principles and the meaning of pseudodelay are explained by means of the Figures similar to Fig. 1 as follows: navigation satellites emit navigation signals at time points \( t_{01}, t_{02}, t_{03}, \ldots \) with an interval of \( T_e \) in the system timescale (STS) (i.e., it is implicitly suggested that the navigation signal is a pulse signal). The time reference generator of the navigation receiver generates consequential time points \( t_{g1}, t_{g2}, t_{g3}, \ldots \) with the same period \( T_e \) defining the time scale of the navigation receiver (RTS). The signals emitted by the satellites at the time points \( t_{01}, t_{02}, t_{03}, \ldots \) are received at the time points \( t_{r1}, t_{r2}, t_{r3}, \ldots \), according to the RTS (i.e., once again a pulse nature of the navigation signal is implied). For convenience, the emission time points \( t_{01}, t_{02}, t_{03}, \ldots \) are connected with the reception moments \( t_{r1}, t_{r2}, t_{r3}, \ldots \) with the inclined dashed arrows in Fig. 1.

In general, the RTS is displaced relative to the STS by a value unknown for the navigation receiver – \( \Delta T \), as shown in Fig. 1 and defined in [4] as \( \Delta T = t_{g0} - t_{g0} \).

The navigation receiver measures the delays of the satellite signals in its scale, i.e. it assumes that the signals are emitted by the satellites at the reference time points \( t_{g1}, t_{g2}, t_{g3}, \ldots \) in the RTS scale, while they are really emitted at the time points \( t_{r1}, t_{r2}, t_{r3}, \ldots \). As a result, the navigation receiver measures not the delay \( \tau_{\rho} \) of the signal propagation from the j-th satellite to the navigation receiver, but the pseudodelay \( \tau_{\rho} \):

\[
\tau_{\rho} = \tau_{\rho} + \Delta T, \quad j = 1, J
\]

where \( J \) is a total number of satellites tracked by the navigation receiver. Therefore, according to Fig. 1, the pseudodelay \( \tau_{\rho} \) in the navigation receiver is formed by measuring the duration of the time interval, which begins at the time points \( t_{r0} \) and ends at the time points \( t_{rj} \).

The pseudodelay (1) multiplied by the light speed \( c \) results in pseudorange \( \rho_{\rho} \)
\[ \rho_j = c t_{pdj} = \rho_{d} + \Delta T = R + c \Delta T = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 + \Delta R_j}, \quad j = 1, J \]

where \(x_i, y_i, z_i\) are the unknown coordinates of the navigation receiver, \(x_j, y_j, z_j\) are the known coordinates of the \(j\)-th satellite acquired from its navigation message, and \(\Delta R_j = \hat{n} \Delta T\) is the unknown RTS displacement relative to the STS in meters.

Pseudodelays \(\rho_j\) measured by not less than four satellites \((J \geq 4)\) are used to make a set of equations (2) with four unknown \(x_i, y_i, z_i,\) and \(\Delta R_j\). The estimates \(\hat{x}_i, \hat{y}_i, \hat{z}_i,\) and \(\Delta \hat{R}_j\) are found from the solution.

2. Critique of the conceptual model used in the modern educational and scientific literature to explain the GNSS functioning principles

The fundamental model given in Section 1, which for convenience is called the old model, employs the terms, the meaning of which is blurred and, at times, senseless. The use of these terms in the old model leads to contradictions. The following several examples prove this statement.

1. In modern GNSS, signals of navigation satellites are continuous periodic pseudorandom sequences (PRS). What is understood in this case under moments of emission and reception of continuous signals, because such signals are emitted and received at any time point, so can any time point be considered the moment of emission and reception?

2. If the pseudodelay \(\tau_{pdj}\) of the signal in Fig. 1 exceeds \(T_c\), the measurement of pseudodelay becomes ambiguous and can be expressed as \(\tau_{pdj} = \tau_{pdj} + k/T_c\), \(j = 1, J\), where \(\tau_{pdj}\) is the ambiguous measurement of the pseudodelay formed by using the one of the time points \(t_{meas1}, t_{meas2}, t_{meas3}, \ldots\), that is the closest and preceding to the moment of reception, as a reference time point in the RTS; \(k\) is an indefinite integer. The same ambiguity appears in a case when the modulus \(|\Delta T|\) of the RTS displacement relative to STS exceeds the period \(T_c\).

Essentially, the ambiguousness of measurements of pseudodelay can be solved with the help of the approximated a priori data on the delay \(\tau_{d}\) and the displacement of \(\Delta T\) RTS relative to STS. At this, the total error of the approximated a priori data on the delay \(\tau_{d}\) and the displacement of \(\Delta T\) should not exceed \(T_c/2\).

By means of such a priori data, using the formula (1) a roughly approximated value \(\tau_{pdj}\) can be calculated. Such rough evaluation of the pseudodelay produces the following approximated equality \(\tau_{pdj} \approx \tau_{pdj} + k/T_c\). The inaccuracy of this equality does not exceed \(T_c/2\). Hence, it is easy to get a formula to calculate an indefinite integer \(k\):

\[ k_j = \left\lfloor \left( \tau_{pdj} - \tau_{pdj} \right)/T_c \right\rfloor, \quad j = 1, J, \]

where \(\left\lfloor x \right\rfloor\) means calculating the integer closest to \(x\). In textbook [7], exactly this method for solving the ambiguousness of measurement of pseudodelay in GNSS is described, though it is not used in any real navigation receivers.

3. According to Fig. 1, measurement of pseudodelay is carried out at the time points of receiving the \(t_{1_1}, t_{2_1}, t_{3_1}, \ldots\) signals emitted at the \(t_{1_0}, t_{2_0}, t_{3_0}, \ldots\) time points. However, measurement of pseudodelay should be conducted simultaneously for not less than four satellites. Because of the difference in the distances between the satellites, the \(t_{1_1}, t_{2_1}, t_{3_1}, \ldots\) time points of receiving signals form the \(j\)-th satellite in the navigation field will not coincide with the time points \(t_{1_1}, t_{2_1}, t_{3_1}, \ldots\) of receiving signals from the \(k\)-th satellite. Therefore, if measurement of pseudodelay for each satellite is carried out at the moment when the signal from this satellite is received, such measurement for different satellites will occur at different time points. What time do the evaluated \(\hat{x}_i, \hat{y}_i, \hat{z}_i,\) and \(\Delta \hat{R}_j\), found from solving the system of linear equations (2), correspond to?

4. A navigational receiver should measure pseudodelays for all the satellites at uniform time points of \(t_{meas}\). It is possible to use, for example, the reference moments \(t_{1_0}, t_{2_0}, t_{3_0}, \ldots\) shown in Fig. 1. The position of these moments on the RTS is defined (is set) by the signal of the generator of the navigation receiver. However, in order to make measurement of the pseudodelays corresponding to different navigational satellites to be carried out in the uniform moments of \(t_{meas}\), it is necessary for the corresponding moments of signal emission from different satellites to differ and precede the moments of measurement \(t_{meas}\) for the period of signals propagation from different satellites to the navigation receiver. Further, for convenience, these time points will be called the preceding moments and will be designated as \(t_{pdj}\), where the superindex \(j\) is the number of the satellite. Time of signal propagation from different satellites can vary depending on the position of the consumer and the altitude of the satellite orbit. Hence, it is clear that
the assumption about the pulse nature of a navigation signal introduced implicitly cannot be accepted, because satellites cannot emit pulses at the time points preceding the moments of measurement in receivers of all the great number of consumers.

To overcome the contradictions of old conceptual model described above, it is necessary to introduce new concepts considered in the following section.

3. Determination of the semantic content of the concepts “timescale” and “time on a scale”

The contradictions of the old conceptual model of radio navigation revealed above cannot be eliminated without determination of the semantic content of the concepts “timescale” and “time on a scale”. Despite the wide usage of these terms in literature [1–14], the author did not manage to find the definition of their semantic content there. Therefore, it is necessary to define the semantic content of the concepts “timescale” and “time on a scale”.

Further, to eliminate the confusion between the terms “time” and “time on a scale”, instead of the term “time”, we shall use the term “physical time” that means the ideal time that lapses absolutely evenly and that is used in physics textbooks. To designate the physical time, the symbol $t$ is used.

The definition of the semantic content of the concepts “timescale” and “time on a scale” demands the definition of the semantic content of a “phase”, as well as the introduction of distinctions in definitions of the semantic content of the concept of phase. Again, despite the wide usage of the term “phase” in literature, the author did not manage to find the definition of its semantic content. In Textbook [16], a mathematical definition of the concept of phase for a harmonic or in a more general case a quasiharmonic process or signal is given

$$a(t) = A(t)\cos\phi(t).$$

Here, $A(t)$ is the slow changing signal amplitude, $\phi(t)$ is the slow changing phase of the signal (in radians), which is an argument of a harmonic function. The argument $\phi(t)$ is determined by the instantaneous angular frequency of the $\omega(t)$ signal by the formula

$$\phi(t) = \int_0^t \omega(x)dx + \phi_0$$

where $\omega(t) = 2\pi f(t)$, $f(t)$ is the instantaneous frequency (in Hz). The first item in the right part (3) is defined as the phase increment on the time interval $0 \to t$, and $\phi_0$ is defined as the initial phase, i.e., the value of the phase $\phi(t)$ at the $t=0$ time moment. The concept of the instantaneous angular frequency $\omega(t)$ is the derivative of phase $\phi(t)$.

$$\omega(t) = \frac{d\phi(t)}{dt}$$

For a strictly harmonic frequency signal, $\omega$ and $f$ are constants, and the phase changes uniformly or linearly: $\phi(t) = \omega t + \phi_0$. In case of a quasiharmonic signal, $\omega(t)$ is a slowly changing function of physical time $t$, and the phase changes nonuniformly. Expressions (3, 4) make it possible to geometrically interpret the phase of a quasiharmonic signal as a vector angle $\phi(t)$ of variable length $A(t)$ rotating with slowly changing instantaneous angular speed $\omega(t)$.

Henceforth we shall use cycle as the more convenient unit of phase. Cycle 1 equals $2\pi$ radian.

The expressions (3) and (4) in this case will be transformed to

$$\phi(t) = \int_0^t f(x)dx + \phi_0, \quad f(t) = \frac{d\phi(t)}{dt}.$$

In practice, there is often a need to consider varieties of the concept of phase, such as fractional and full phase. The fractional phase $\phi_{\text{frac}}(t)$ is the phase lying within 1st cycle $0 \leq \phi_{\text{frac}} < 1$. The full phase $\phi_{\text{full}}(t)$ can accept any values, i.e., contain besides a fractional phase $\phi_{\text{frac}}(t)$ in its structure the integer number of cycles $n(t)$ counted at every moment of physical time $t$ from a starting point defined in advance.

$$\phi_{\text{full}}(t) = \phi_{\text{frac}}(t) + n(t)$$

When measuring a phase there can be situations when the integer $n(t)$, which is a part of the full phase (5) differs from its true value by an uncertain number of cycles. Such full phase is called an ambiguously full phase.

A cyclic interval is an interval of physical time $t$, during which the full phase goes up by 1 cycle. In case of uneven change of phase, cyclic intervals will have various duration.

A fractional phase of a signal can be deduced from the full phase by adding or subtracting such an integer number of cycles for the result to be ranging from 0 to
1 cycle. It is known that addition of an integer number of $2\pi$ (an integer number of cycles) to the argument of the harmonic function does not change the value of this function. In this case, full and submultiple phases are equivalent to each other.

The concept of phase is applicable not only to harmonic or quasiharmonic signals. Fig. 2(b) shows change in time of the pseudorandom sequence (PRS) 11110 00100 11010 at uneven change of its phase, and Fig. 2a shows the schedule of this unevenly changing phase.

Fig. 2. PRS with uneven phase changing

Fig. 2b shows two identical in structure PRS 11110 00100 11010, located on cyclic intervals of physical time that are different in duration. Each of these cyclic intervals begins and ends at the time of the pulse leading edge corresponding to the first one in the group of four ones in a row in the PRS structure. The phase increment of these PRS on these different cyclic intervals is identical and equal to one cycle ($2\pi$ rad).

The example shows that in order to define the concept of signal phase, it is necessary to distinguish the concepts of time period and structural period of a signal. Usually, a time period is thought of as a periodically repeating strictly identical interval of physical time. It is necessary to understand that an interval of physical time, on which all structure elements of a signal repeat, is a structural period. This period can have variable duration, but the signal phase increment on it is always equal to 1 cycle. In case of uniformly changing phase, the time intervals, on which the phase increment increases by 1 cycle, become identical, and then the concepts of structural and time periods coincide.

Based on the concept of structural period of a signal, the concept of fractional phase $\phi_{\text{frac}}(t)$ of this signal in cycles is possible to define as the fraction of its structural period (cycle) observed at every moment of physical time $t$. A full phase of a signal is defined as a sum of the integer number of structural periods (cycles) and the fractional phase of the current structural cycle, which are observed on an interval from the beginning of the count of physical time until the present moment $t$.

In practice, determination of a quantitative value of physical time $t$ is always carried out by means of a clock, which is understood to be a set of means and actions aimed to determine a quantitative value of physical time as a full phase of some periodically repeating process, which is the foundation of the specified clock. Oscillations of a pendulum, a signal of an electric generator, rotation of the Earth or radiation of atoms when they transit between different energy levels that defines an atomic time, can be used as such process. Hereafter, the process or a signal, which is the underlying operation principle of a clock, will be called a process or a signal of this clock.

A quantitative determination of physical time $t$ will be understood as determination of a number for each its moment $T(t)$ that is the value of time at this moment. The specified number $T(t)$ will be called the readings of the corresponding clock for the moment of physical time $t$ under consideration.

Different clocks have different accuracy. The accuracy of a clock is determined by stability of the process of this clock. Therefore, there is a need to distinguish from the known natural processes the most stable one and use the readings of the clock built on its basis as the reference time. According to the present international agreements, the radiation of a cesium atomic beam standard is used as the process of the master clock. By definition, a second as unit of physical time equals 9192631770 periods of radiation corresponding to transition between two super thin levels of the main condition of a cesium-133 atom. However, if one compares the readings of two master clocks using...
different instances of the device counting the radiation periods of -133 atoms, it becomes evident that in course of time these clocks begin to disagree. This happens because any periodic process used for determination of a quantitative value of physical time has instability and this instability leads to the fact that in course of time even very precise clock disagree. Therefore, readings $T(t)$ of any clock are only approximations to what physical time $t$ is.

Readings of any clock are formed as the sum of their initial setting, the number of full phase increments of the clocks process on an interval of physical time from the moment of the initial setting to the present moment and possible corrections of the clock readings on the same interval of physical time. If readings of a clock are measured in seconds, then on the time interval from the moment of the initial setting a quantitative increment of time is defined as an increment of the full phase of the clock process brought to 1 Hz. This means that the increment of a full phase of the clock process divided by the nominal value of frequency of this process. For example, the increment of a full phase of radiation of the cesium atomic beam standard brought to 1 Hz is defined as the number of increments of the full phase of this radiation divided by 9192631770.

Therefore, readings of clock $T(t)$ is a phase, the value of which is used for the quantitative measurement of physical time. At the time of taking of a clock’s readings (i.e., at the time of measurement of the quantitative value of physical time) the phase is treated as time, and the unit of measurement of phase is replaced with the unit of measurement of time.

We shall define the concept of a timescale as moments of physical time $t$ set by the readings of the clock, which are the basis of the scale under consideration [17, 18]. Then, the concept of time on a scale is defined as the readings of the clock, which are the basis of the scale for any moment of physical time $t$. At the same moment of physical time different clock can have different readings (different time on different scales) and at the different moments of physical time different clock can have identical readings (identical time on different scales). A timescale shift should be understood as the difference of the readings of a clock on one scale and a clock on another one at the same moment of physical time. At the same time, the difference of the clocks readings for the same moment of physical time should not be confused with an interval between the moments of physical time, at which the clock readings are identical. Since any clock is unstable, difference of the clock readings for the same moment of physical time generally is not equal to time interval between the moments of identical readings of this clock.

4. Description of the GNSS functioning principles based on the new conceptual model

The signals emitted by navigation satellites in the modern GNSS are the high frequency phase-modulated carrier oscillations in the range ~1.2–1.6 GHz. Modulation of the carrier oscillations is carried out by a double-layer signal. The lower layer is a continuous periodically repeating PRS, on which measurement of pseudoranges is carried out. A nominal period of these PRS in the open signals of GLONASS and GPS is equal to 1 ms. The upper layer is formed by binary 20 ms symbols of the navigation message, which inversely modulate periodically repeating PRS of the low layer. Formation of PRS in the onboard equipment of satellites is carried out from a signal of the high-stable atomic frequency standard. A full phase of PRS, emitted by each satellite and interpreted as the readings of clock, sets the onboard time scale (OTS) of this satellite.

According to the definition (5), a full phase $\phi_{\text{full}}(t)$ of PRS for each present moment of physical time $t$ is set by a fractional phase $\phi_{\text{frac}}(t)$ by this PRS and an integer $n(t)$ of the full periods of PRS, which are keeping within an interval from some conditional beginning defined in advance before the present moment $t$. For example, in GLONASS system such conditional beginning are 00 hours, 00 min 00 sec from January 1, 1996, according to the Moscow standard time defined as UTC (SU)+3 hours. For setting an integer number of cycles $n(t)$, special signals of timestamps and digitization of these timestamps $\zeta_j$ are put in navigation messages of satellites. A signal of a timestamp is an a priori defined sequence of pulses in the navigation message. The moment of emergence of the trailing or front edge of a certain pulse in the signal of a timestamp is the timestamp itself. Further, this moment will be called a timestamp moment. For example, in the GLONASS system, a timestamp moment is the moment of the trailing edge of the last pulse of a signal of a timestamp, and in GPS, a stamp moment is the moment of the forward front of the first pulse of a signal of a timestamp. Digitization of a timestamp moment is
the readings of clock of the j-th satellite on its board at this moment. Fig. 3 shows characteristic time points in the transmitted (Fig. 3a) and received (Fig. 3b) signal. In Fig. 3a, the moments of the beginning of the periods of PRS in the radiated signal, or in other words the moments of milliseconds according to RTS, are shown with the arrows pointed up. A timestamp moment is italicized with a big arrow. The symbol $\zeta^j_\text{sec}$ shown over a big arrow designates digitization of this timestamp moment. The corresponding time points in the received signal are shown in Fig. 3b in the form of hyphens with crosses. The hyphens focused down on Fig. 3b show some in general case random timestamp moments of RTS some generally. In general, it is not supposed that these time points have any digitization.

In the moment of time on RTS marked in Fig. 3b with a symbol $t_{\text{meas}}$, measurement of a fractional phase $\hat{\xi}^j(t_{\text{meas}})$ of PRS of the j-th satellite is taken in the navigation receiver. This phase expressed in cycles is equal to the period share $b/a$ in the received signal, which has passed from the beginning of the PRS period until physical time $t_{\text{meas}}$. The value of $\hat{\xi}^j(t_{\text{meas}})$ cannot be displayed in Fig. 3, as for this purpose it is necessary to allow a vertical axis, along which the phase (the reading of clock) will be laid off. Such laying off will be made further in Fig. 5 in the form of the clock readings.

As it is seen from Fig. 3, the value of a fractional phase $\hat{\xi}^j(t_{\text{meas}})$ measured in the receiver with an accuracy up to an integer of milliseconds and errors of tracking is in agreement with the readings of satellite clock at the time of precedence $t^j_r$ to the measurement moment $t_{\text{meas}}$. The assessment $T^j_r(t^j_r)$ of the complete clock readings of the j-th satellite in seconds at the time of precedence is calculated in the processor of the navigation receiver according to the formula

$$\hat{\xi}^j(t_{\text{meas}}) = 10^{-3}(\zeta^j_{\text{sec}} + n^j + \hat{\xi}^j(t_{\text{meas}})),$$

where $\zeta^j_{\text{sec}}$ is the digitization of the last accepted timestamp expressed in milliseconds; $n^j$ is the whole amount of the periods of the accepted PRS lying on a time interval from the last accepted and digitized timestamp until measurement $t_{\text{meas}}$ (in the example, shown in Fig. 3. n = 2). The actions described above and the funds allocated for this purpose for estimation of the clock readings of the j-th satellite at the time of precedence can be called channel clock of the j-th satellite in the navigation receiver, and the estimates determined by a formula (6) are called readings of this clock. For convenience of the further consideration of the channel clock reading, relating to the time of measurement $t_{\text{meas}}$, will be designated as $T^j_{\text{chan}}(t_{\text{meas}})$, i.e., $T^j_{\text{chan}}(t_{\text{meas}}) = T^j(t^j_r)$. It is possible to interpret the calculations by a formula (6) as a solution of millisecond ambiguity of estimates $\hat{\xi}^j(t_{\text{meas}})$ of the readings of satellite clock.

The channel clock under consideration is schematically shown in Fig. 4 with four small circles. It is obvious that the number of channel clock of the navigation receiver is equal to the number of its channels.

Apart from the channel clocks in the navigation receiver, its own clock, which is schematically shown in Fig. 4 with a lower big circle, is used. Own clock of the receiver is the clock, which readings define the moments of carrying out measurements, i.e., sets a receiver time scale.

The coordinates of the navigation receiver and the clock reading of the system for a moment $t_{\text{meas}}$ can be determined based only on the readings of the channel clock. The receiver derives the values of the polynomial
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models coefficients from navigation messages allowing one to calculate the estimates of shifts $\Delta \tilde{T}_{sys}^j(t_p^j)$ of the clock readings of all tracked satellites relative to the clock readings of the system at the time of precedence $t_p^j$. Further, by means of these estimates, the receiver calculates the estimates of the clock readings of the system for the precedence moments:

$$\tilde{T}_{sys}^j(t_p^j) = \tilde{T}^j(t_p^j) - \Delta \tilde{T}_{sys}^j(t_p^j),$$

$$j = \overline{1, J}$$

(7)

Using the parameters of mathematical models of satellites movement, transferred in navigation messages and the estimates $\tilde{T}_{sys}^j(t_p^j)$, the receiver calculates the coordinates $x^j(t_p^j)$, $y^j(t_p^j)$, and $z^j(t_p^j)$ of each j-th satellite for the precedence moment corresponding to this satellite. It should be emphasized that in order to calculate the coordinates of each j-th satellite, the receiver uses not the value $t_p^j$ of physical time for the precedence moment, but the assessment $\tilde{T}_{sys}^j(t_p^j)$ of the clock readings of the system for this moment, or, otherwise, to calculate the navigation satellites coordinates in GNSS, it is necessary to have not the value of physical time, but the assessment

$$\tilde{T}_{sys}^j(t_p^j)$$

of the readings of own clock of the receiver at the moment of precedence $t_p^j$, at the time of measurement $t$, from (10), the following system of the nonlinear equations concerning unknown $T_{sys}(t_{meas})$, $x_r(t_{meas})$, $y_r(t_{meas})$, $z_r(t_{meas})$ is received:

$$T_{sys}(t_{meas}) - \frac{R^j}{c} = \tilde{T}^j(t_p^j) - \Delta T_{sys}^j(t_p^j),$$

$$j = \overline{1, J}$$

(10)

Expressing in (10) the distance $R^j$ through the coordinates of the navigation receiver $x_r(t_{meas})$, $y_r(t_{meas})$, $z_r(t_{meas})$, at the time of measurement $t_{meas}$ and the coordinate of the j-th satellite $x^j(t_p^j)$, $y^j(t_p^j)$, $z^j(t_p^j)$, at the moment of precedence $t_p^j$, from (10), the following system of the nonlinear equations concerning unknown

$$T_{sys}(t_{meas}), x_r(t_{meas}), y_r(t_{meas}), z_r(t_{meas})$$

is obtained:

$$T_{sys}(t_{meas}) - \frac{1}{c} \sqrt{(x_r(t_{meas}) - x^j(t_p^j))^2 + (y_r(t_{meas}) - y^j(t_p^j))^2 + (z_r(t_{meas}) - z^j(t_p^j))^2} = \tilde{T}^j(t_p^j) - \Delta \tilde{T}_{sys}^j(t_p^j),$$

$$j = \overline{1, J}$$

(11)

To find four unknown of $T_{sys}(t_{meas})$, $x_r(t_{meas})$, $y_r(t_{meas})$, $z_r(t_{meas})$, it is necessary to have not less than four equations of a type (11), i.e., to carry out measurements at the same time not less than on four satellites. Solving a system (11) under these conditions, the estimates $\hat{x}_r(t_{meas})$, $\hat{y}_r(t_{meas})$, and $\hat{z}_r(t_{meas})$ of the coordinates of the navigation receiver and the assessment

$$\hat{T}_{sys}(t_{meas})$$

of the clock readings of the system at the time of measurement $t_{meas}$, which can be used further as digitization of time moment, are obtained.

In the system (11), own clock readings of the navigation receiver are not used, i.e., it is not required that timestamps of the receiver should be digitized. However, in practice, usually it is required to carry out navigation definitions not in randomly set measurement moments $t_{meas}$, but in regular intervals. To count these intervals, it is necessary to use own clock of the navigation receiver shown in Fig. 4 with a big circle. In this case timestamps of the navigation receiver are digitized by readings of its own clock, and instead of the readings of channel clock, a concept of a pseudodelay is employed. At the same time, it is unimportant, how precisely these digitizations coincide with the clock readings of the system $T_{sys}(t_{meas})$ at the same moment $t_{meas}$. The pseudodelay

$$\tau_p(t_{meas})$$

according to the j-th satellite is determined as a difference of the readings of own clock of the receiver $T_r(t_{meas})$ at the time of measurement $t_{meas}$ and the clock readings of the j-th satellite at the time of precedence $t_p^j$:
The initial value $T_j(t_{meas})$ can be set randomly, taken from any suitable source, or just calculated according to the following approximate formula:

$$T_j(t_{meas}) = \zeta_j + 0.8 \ c$$

(13)

where $\zeta_j$ is digitization of the next accepted timestamp from any satellite. The error of initial digitization of timestamps of the receiver by a formula (13) does not exceed $\pm 30$ ms.

A pseudodelay assessment $\hat{\tau}_j^i(t_{meas})$ formed in the receiver is defined as a difference of the readings of own clock of the receiver and the readings of its channel clock corresponding to the j-th satellite at the time of measurement $t_{meas}$:

$$\hat{\tau}_j^i(t_{meas}) = T_j(t_{meas}) - T_{\text{chan}}(t_{meas}) = T_j(t_{meas}) - \hat{T}_j^i(t_p^j),$$

(14)

where the symbol $t_p^j$ in that case designates the precedence moment to a present situation of physical time $t$. For any moment of this time, it is possible to introduce the concepts of shifts of the clock readings of the satellite and own clock of the receiver:

$$\Delta T_j^i(t) = T_j(t) - T_{\text{sys}}(t), \quad j = 1, J$$

(15)

$$\Delta T_j(t) = T_j(t) - T_{\text{sys}}(t)$$

(16)

Using (16), the clock readings of the j-th satellite and the reading of own clock of the receiver $T_j(t)$, the following can be expressed through the shifts:

$$T_j(t) = T_{\text{sys}}(t) + \Delta T_j^i(t), \quad j = 1, J$$

$$T_j(t) = T_{\text{sys}}(t) + \Delta T_j(t)$$

(17)

Substituting (17) into (12), the following expression for pseudodelay is obtained:

$$\tau_{pd}^j(t_{meas}) = T_j(t_{meas}) - T_j^i(t_p^j) =$$

$$= T_{\text{sys}}(t_{meas}) - T_{\text{sys}}^i(t_p^j) + \Delta T_j(t_{meas}) - \Delta T_j^i(t_p^j) =$$

$$= \Delta T_j^i(t_{meas}) + \Delta T_j(t_{meas}) - \Delta T_{\text{sys}}^i(t_p^j)$$

(18)

where

$$\Delta T_{\text{sys}}^i(t_p^j + t_{meas}) = T_{\text{sys}}(t_{meas}) - T_{\text{sys}}(t_p^j)$$

(19)

is increment of the clock readings of the system on the time interval $t_p^j + t_{meas}$, duration of which is equal to the delay $\hat{\tau}_j^i(t_{meas}) = t_{meas} - t_p^j$ of a signal propagation from the point occupied by the j-th satellite in the preceding moment $t_p^j$ until the point occupied by the receiver in the moment of measurement $t_{meas}$.

Fig. 5 shows the pseudodelays change as the functions of physical time $t$ for two satellites with Nos. j and k.

A pseudorange assessment $\hat{\rho}_j^i(t_{meas})$ according to the j-th satellite is defined as a pseudodelay assessment (14) multiplied by light speed $c$:

$$\hat{\rho}_j^i(t_{meas}) = c \cdot \hat{\tau}_j^i(t_{meas}) =$$

$$= c \left( T_j(t_{meas}) - T_{\text{chan}}(t_{meas}) \right), \quad j = 1, J$$

(20)

From (14) and (20) it is easy to see that at strict synchronism of the clock rate of the receiver and satellites, a pseudodelay assessment becomes a delay assessment, and a pseudorange assessment turns into a range assessment.

Subtracting the readings of own clock of the receiver $T_j(t_{meas})$ from the left and right parts (11), the following is received:
By the definition (16), the contents of the parentheses standing in the left part of the expression (21) is a shift of the clock readings of the receiver concerning the clock readings the system at the time of measurement. The following should be introduced to the product of this shift and light velocity $c$:

$$\Delta R_r(t_{\text{meas}}) = \hat{\delta}^j(t_{\text{meas}}) + c \cdot \Delta T^j_{r}\left(t_{pr}\right),$$

where $\Delta R_r(t_{\text{meas}})$ is a shift of the readings of the receiver clock concerning the readings of the system clock expressed in meters. From (22) it is seen that it is possible to calculate the assessment $\hat{T}_{\text{sys}}(t_{\text{meas}})$ of the readings of the system clock at the time of measurement $t_{\text{meas}}$, as $T_{\text{sys}}(t_{\text{meas}}) = T_r(t_{\text{meas}}) - \Delta \hat{R}_r(t_{\text{meas}}) / \hat{a}$ by means of a shift assessment $\Delta \hat{R}_r(t_{\text{meas}})$ and the readings of the receiver clocks $T_r(t_{\text{meas}})$.

### 5. Application of the new conceptual model to ground radio navigational systems

Ground RNS can be considered as a simplified GNSS option. Simplification is that transmitters of navigation signals in these systems are fixed and, therefore, the coordinates of transmitters can be placed in memory of navigation receivers during their production. Timescales of all transmitters of ground RNS, as well as in GNSS, are synchronized with high precision with a system timescale. The structures of the transmitted navigation signals in various ground RNS can differ greatly, but the main principle remains invariable: the phases of the radiated radio navigation signals carry information on the readings of the system clock and set time scales of the received signals in navigation receivers.
It is not possible to consider all types of ground RNS in the article. Therefore, further, as an example, the application of a new conceptual fundamental for the ground super long-wave RNS OMEGA [15] will be considered.

The structure of a navigation frame of the RNS OMEGA is shown in Fig. 6. [4, 15, 19].

Eight stations of the RNS OMEGA are given in the Latin letters A, B, C, D, E, F, G, and H, which with time shift emit radio pulses with an average duration of 1.25 sec., filled with coherent harmonic oscillations at frequencies of 10.2, 13.6, and 34/3 kHz respectively. Navigation radio pulses are highlighted in Fig. 6 with dotted pattern. Radio pulses with unique frequencies used for stations identifications are shown in vertical hatch in Fig. 6. The rest four radio pulses of each line of the navigation frame are used for exchange between the stations [19].

The main navigation frequency in the RNS OMEGA is 10.2 kHz, i.e., a signal phase of this frequency is identified with the readings of the synchronically working clock of the system stations (system clock). However, a signal period of frequency 10.2 kHz equal to 0.09804 ms is very small and, consequently, a fractional phase of this frequency measured in the navigation receiver, carry information on the readings of the system clock of the moment of precedence with an accuracy up to an integer of periods 0.09804 ms. In other words, the readings of the system clock of the precedence moments identified with a fractional phase of frequency 10.2 kHz are measured in the navigation receiver ambiguously. To solve this ambiguity, one applies measurement of fractional phases on difference frequencies: 13.6–10.2 = 3.4 kHz (period of 0.294117 ms) and 34/3–10.2 = 34/30 kHz (period of 0.88235 ms) [4, 15]. The least common multiple of these periods is 60/17 ms, i.e., a time interval, through which fractional phases of harmonic signals of all navigation radio pulses transform into null, equals 60/17 ms [15].

Time moments following the system clock through 60/17 ms are called the RNS OMEGA eras. If measurement of a signal phase of a navigation frame, which has duration of 10 sec (see Fig. 6), is involved to solve ambiguity, then a signal phase at a frequency of 10.2 kHz can be unequivocally measured in the navigation receiver within 30 sec. Solving 30 sec ambiguity is possible by means of normal clock.

It should be noted that in literature [4, 15] when stating the methods of ambiguity solving in the RNS OMEGA, it is said that either ambiguity of range measurement or delays of a signal is solved. Obviously, there is a question about the kind of a range or delay of a signal, if in the noninterrogative systems, which the RNS OMEGA belongs to, a range or delay of a signal cannot be fundamentally measured. Actually, just as in GNSS, ambiguity not of range, but the readings of the clock of the RNS OMEGA stations for the precedence moments is solved. Taking into account that the clock of the stations is synchronized with the system clock, it is possible to speak about disambiguation of the readings of system clock for the precedence moments. The readings of the stations clock is transferred continuously in the phases of harmonic carriers, filling navigation radio pulses. Nevertheless, in literature [4, 15], the concepts of the moments of precedence and the readings of the stations and system clock are not introduced. In addition, it is not specified that phases of harmonic carriers, filling the emitted navigation radio pulses, carry the information on the readings of the stations clock in the precedence moments to the moments of phases measurement of these carriers in navigation receivers. As a result, the concept (the readings of the stations clock for the precedence moments), which ambiguity of measurements is solved, in literature [4, 15] is absent. For this reason, the authors [4, 15] are forced to speak about disambiguation of range measurements.

In addition, in literature [15], it is specified that for disambiguation of pseudoranging measurements, it is necessary to have a priori data not only on the coordinates of the navigation receiver, but also on shift of its timescale. The necessity in having a priori data for solving disambiguation of measurements of pseudoranges in the RNS OMEGA, as well as the need in having such data for solving disambiguation of measurements of pseudoranges in GNSS, about which the manual [7] states, is a mistake. This mistake results from desire of the authors of literature [7, 15] to solve ambiguity of measurements of pseudoranges. Such desire is natural, as in [7, 15] the concepts of the moments of precedence and the readings of the stations (satellites) clock for these moments are not used. However, if to solve ambiguity not of pseudoranges, but that of the readings of the stations (satellites) clock for the precedence moments (that, actually, is done in GNSS), then no a priori data in GNSS are required. In the RNS OMEGA, only rough a priori data on the readings of the system clock for the precedence moments will be required. The errors of these
rough a priori data on the module should not exceed 15 seconds. A priori data with such big errors can be received by means of the regular clock, which is periodically set at the signals of the exact time, being broadcast.

In GNSS, disambiguation of the clock readings of satellites is carried out using timestamps and their digitizations (see Section 4 of this article). In the RNS OMEGA, digitization of timestamps is absent, and the moments of the beginning of 30 sec intervals corresponding to the moments of the RNS OMEGA eras can be used as timestamps. On each such interval, there are three in succession frames. As it was shown earlier, using the measurements of fractional phases on the main and difference frequencies and the readings of external clock, which shift concerning the readings of the system clock does not exceed 15 sec, permits one to solve completely the ambiguity of clock readings of the stations during the precedence moments. After disambiguation solving, unequivocal values of pseudotime delays as differences between the clocks readings of the navigation receiver at the time of measurement and the solved clock readings of the stations within the precedence moments can be formed. At the same time, it is no matter how much a time scale of the navigation receiver is offset concerning a system scale.

If digitizations of timestamps are introduced into the structure of the RNS OMEGA navigation signal, so in this case to solve disambiguation of clock readings of the stations, as well as in GNSS, no a priori data will be required. The paper considers [19] the offers on digitizations of timestamps into the structure of the RNS OMEGA navigation frame. For this purpose, it is offered to introduce a concept of five-minute superframes shown in Fig. 7. In the superframe, the first two minutes are separated for signal transmission of a timestamp designating the beginning of a superframe and digitization of this stamp. The last 3 minutes of a superframe are separated for transfer of codes of interstations exchange.

Each line of the RNS OMEGA navigation frame (see Fig. 6) includes eight radio pulses, four of which were not used for any purposes at the time of publication [19]. These four radio pulses in work [19] are offered to use for transmitting one tenth of a figure by means of a binary code. Unites and nulls of a binary code are offered to transmit via radio pulses frequency change. To do this, two individual frequencies are given for each RNS OMEGA station. On the two-minute time interval, given for transmitting a signal of a timestamp and its digitization, there are 12 frames. Hence, it is possible to transmit 12 decimal digits on this interval. A signal of a timestamp designating the beginning of a superframe is transmitted in the first frame of a superframe. A number of a minute in the hour is transmitted in the frames Nos. 2 and 3. A number of an hour per day is transmitted in the frames Nos. 4 and 5. Three frames Nos. 6, 7, and 8 are separated for transmitting a number of a day of a year. It is proposed to transmit a number of a year of a century in the frames Nos. 9 and 10. Usage of the rest frames Nos. 11 and 12 is not determined. The structure of a two-minute time code of the RNS OMEGA offered in [19] is shown in Fig. 8.
If timestamps of the signals, radiated by the RNS OMEGA stations, are considered as the moments of the beginning of a 30-seconds intervals coinciding with the moments of the RNS OMEGA eras, so each 10th stamp will be digitized. At such digitization, solving the disambiguity of the clock readings of the stations, counting down in seconds from the beginning of the current year (i.e., with an accuracy up to a whole number of years from the century beginning) should be calculated according to the formula similar to (6)

$$\hat{T}_j\left(t^r_p\right) = N_{d} \cdot 86400 + N_h \cdot 3600 + N_{\min} \cdot 60 + N_{30} \cdot 30 + \xi_j\left(t_{\text{meas}}\right), \quad j = 1, J$$

where $N_{d}$ is an amount of days in a year finished up to the beginning of the current superframe; $N_h$ is an amount of hours in the current year finished up to the moment of the current superframe (the values $N_{d}$, $N_h$, $N_{\min}$ are separated from the received superframe); $N_{30}$ is an amount of 30-minutes intervals from the beginning of a current superframe finished up to the moment $t_{\text{meas}}$ of carrying out the measurements (the value $N_{30}$ is calculated in the receiver); $\xi_j\left(t_{\text{meas}}\right)$ is a phase of the received signal at the frequency 10.2 kHz unequivocally expressed within 30 sec, referring to 1 Hz (the value $\xi_j\left(t_{\text{meas}}\right)$ is determined via solving the disambiguity of the measured value of the fractional phase of a signal frequency 10.2 kHz using the measurements of fractional phases at difference frequencies [4, 15]).

Subtracting the value $\hat{T}_j\left(t^r_p\right)$ calculated according to the formula (24) based on the clock readings of the receiver $T_{s}\left(t_{\text{meas}}\right)$ in the moment of the measurement $t_{\text{meas}}$, a univocal value of pseudodelay corresponding to the j-th station of the RNS OMEGA is given. It is possible to offer one more method (an easier one) to solve the disambiguity of measurement of a fractional phase of a signal frequency 10.2 kHz to resume a whole number of years from the century beginning (i.e., with an accuracy up to a whole number of years from the beginning of the century) can be carried out with a formula similar to the formulae (6, 24).

$$\hat{T}_j\left(t^r_p\right) = N_{d} \cdot 86400 + N_h \cdot 3600 + N_{\min} \cdot 60 + N_f \cdot 10 + \frac{N_{10} + \xi_j\left(t_{\text{meas}}\right) \cdot 10^{-3}}{10.2}$$

It should be noted that in (25) for solving the disambiguity $T^r(t^r_p)$, the measurements of fractional phases on the measurement frequencies 13.6 and 34/3 kHz are not used. Thus, applying a new fundamental concept of readings $T^r(t^r_p)$ of stations clock during the moments of precedence $t^r_p$ enables one to abandon carrying out measurements on the frequencies 13.6 and 34/3 kHz and, hence, ease significantly the RNS OMEGA by introducing digitization of timestamps into the structure of the RNS OMEGA navigation signal and two additional counters into the equipment of the receiver.

The analysis being carried out shows that the concepts introduced in the Sections 3-4 for GNSS are fully acceptable for the ground RNS OMEGA.

Conclusions

Based on the critical review performed in the Sections 1 and 2 of a new conceptual model offered in the Sections 3 and 4, as well as, applying the concepts of a new model for the ground RNS, it is possible to conclude the following:

1. A timescale and clock readings scale of RNP (time according to the RNP scale) in the precedence moments are considered the key concepts of radio navigation. A timescale is the moments of physical time defined by the clock readings that are the base of any scale. Time according to the scale in any moment of physical time is defined as the readings of this clock. Clock is understood as a combination of methods and
actions directed to defining a quantitative value of time according to the scale as reducing to 1 Hz of a complete phase of the periodic process that is a base of a timescale.

2. A concept of pseudodelay (pseudorange) is secondary with regard to the concept of the clock readings (time according to the scale) due to the following:

- A concept of pseudodelay \( \tau_p(t_{meas}) \) is determined through the concept of the clock readings as a difference between the readings \( T_p(t_{meas}) \) of the clock of a navigational receiver in the moment of measurement \( t_{meas} \) and the measurements \( T'_p(t'_p) \) of the clock of the j-th station (satellite) in the precedence moment \( t'_p \) to the moment of measurement \( t_{meas} \). i.e. \( \tau_p(t_{meas}) = T_p(t_{meas}) - T'_p(t'_p) \). The clock readings are an independent concept. As it is shown in the Section 4 of the paper, the evaluations of the clock readings of the GNSS satellites in the precedence moments (the readings of a channel clock in the measurement moments) make it possible to carry out all navigation definitions without using a pseudodelay concept.

- In the synchronism mode in the registers of the phases of reference signals, a tracking loop for the phases of the received signals in the navigation receiver, disambigual evaluations of the stations (satellites) in the moments of precedence are formed. That means that tracking loops of the navigation receiver track not the values of pseudodelays, but the stations (satellites) readings in the precedence moments. The measurements of pseudodelays in the navigation receiver are formed on the secondary base by means of the integration of the receiver’s processor codes of its tracking loops.

- In the RNS, solving the disambiguity of pseudodelay measurements is undertaken not directly, as it is stated in [7, 15], but through solving the disambiguity of the clock readings of the stations (satellites) in the precedence moments. To solve this disambiguity, the signals of timestamps and their digitization are employed. At this, to solve the disambiguity no any a priory information is applied. When there are no time stamps digitization in the radiated signals, it is necessary to use a priory information in the form of the readings of the external clock in the precedence moments.

3. Applying a new fundamental concept of the readings \( T'_p(t'_p) \) of the stations clock during the precedence moments \( t'_p \), permits one to abandon carrying out measurements on the frequencies 13.6 and 34/3 kHz and, hence, ease significantly the RNS OMEGA by introducing digitization of timestamps into the structure of the RNS OMEGA navigation signal and two additional counters into the equipment of the receiver.

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