Determination of the Relative Position of Objects by the First Phase Measurment Differences of One Epoch

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Abstract. A new method is considered that allows determination of the relative position of objects (the vector of the baseline) within a millimeter error by the fractional parts of the first differences in the phase measurements of one epoch. It is shown that the unknown coordinates of the end of the baseline vector correspond to the basic minimum of the reduced quadratic function. An algorithm for searching for local minima has been developed, as well as two approaches to selection of the main minimum: decision-making by the threshold and decision-making by the absolute minimum. An algorithm for computing the baseline vector is given and probabilistic and time characteristics are given for its implementation for the case of sharing single-frequency (L1) range of GLONASS and GPS measurements.
The method is called the "RSS method" by the name of the patent holder (JSC «Russian Space Systems»). A classification of known methods for resolving the ambiguity of phase measurements is presented, which includes the RSS method. The RSS method is a geometric method, in which the search for spatial coordinates of the end of the baseline vector is performed in a geocentric coordinate system with the elimination of the unknown integer number of phase cycles. The method is insensitive to the loss of the count of the phase cycles of the received signals.

Keywords: global navigation satellite systems, relative positioning, phase measurements, disambiguation.

Introduction

Global navigation satellite systems (GNSS) are increasingly used in solving various tasks for both military and civilian purposes. The most accuracy when using GNSS can be achieved when determining the relative position of objects. Relative position of objects with high accuracy is required in geodesy, during construction, monitoring of displacements of engineering structures and the earth's surface, unmanned control of aircraft and ground vehicles, etc. [1]. It is also used for spatial orientation of moving objects and mechanisms.

The relative (mutual) position of objects can be defined by the vector $\bar{L}$ of the baseline, the origin of which is at the point 1 with the coordinates $\{x_1, y_1, z_1\}$, and the end is at point 2 with the coordinates $\{x_2, y_2, z_2\}$: $\bar{L}(x_2 - x_1; y_2 - y_1; z_2 - z_1)$. In this case, the user does not need the absolute coordinates of the points, or they are known with an allowable error.

When using GNSS, the vector $\bar{L}$ is determined from the differences in the measurements of navigation receivers installed at points 1 and 2. Code measurements make it possible to provide meter accuracy, and using phase measurement differences opens the possibility of determining the relative position of objects with a millimeter error. In the latter case, the first and second phase measurement differences are usually used.

The main problem in the processing of phase measurements is their ambiguity associated with the cyclic nature of the phase. A large amount of scientific works has been devoted to the problem of resolving phase ambiguity for the tasks of positioning objects using GNSS. The monograph [2] based on [3] and [4] contains a classification of methods for resolving the ambiguity of phase measurements. This classification is shown in Fig. 1, where references are also made to the literature in which the methods are described.

The method considered in this article will be called, for brevity, "the RSS method" by the name of the organization (JSC "Russian Space Systems"), which received a patent for it [12]. It is a geometric method in which the search for spatial coordinates of the end of the vector of the baseline is performed in the geocentric coordinate system. A distinctive feature of the RSS method is the elimination of the unknown whole cycles in phase measurements and the determination of the coordinates of the end of the baseline vector along fractional parts of these measurements at each epoch at the rate of measurement. This gives it a number of significant advantages. In particular, it becomes insensitive to the loss of counting of whole phase cycles. The RSS method, instead of filtering phase measurements (ambiguous in nature), performs filtering (smoothing) of the calculated coordinates. This can be very effective for dynamic objects with a relatively weak power potential of the radio link or in conditions of poor electromagnetic environment.

Formulation of the problem

We assume that at points 1 and 2, at moments t_{r1} and t_{r2}, the navigation signals of two satellite constellations, such as GPS and GLONASS in the L_1 frequency band, are received. As a result, at the output of receivers for one epoch we will have n measurements of pseudorange by the code of the pseudo-random sequence and the complete pseudo phase of the carrier frequency of the navigation signal: ρ_1^j and Φ_1^j for the first receiver and ρ_2^j and Φ_2^j for the second receiver. Here and below, the indices $j = 1, \ldots, n_{GPS}$ will refer to the GPS satellites, and the indices $j = n_{GPS} + 1, \ldots, n$ - to the GLONASS satellites.

The measured pseudorange values are related to the true ranges R_1^j and R_2^j by the relations:

$$\rho_1^j = R_1^j + cT_1^j + \xi_1^j;\ \rho_2^j = R_2^j + cT_2^j + \xi_2^j,\ j = 1, \ldots, n,$$

where

$$R_1^j = \sqrt{(x^j - x_1)^2 + (y^j - y_1)^2 + (z^j - z_1)^2}, \quad (1)$$

$$R_2^j = \sqrt{(x^j - x_2)^2 + (y^j - y_2)^2 + (z^j - z_2)^2}, \quad (2)$$

$\{x^j, y^j, z^j\}$ - is the coordinates of the *j*-th satellite,
c is the speed of light,

T_1^j *and* T_2^j are the displacements of the time scales of the first and second receivers from the system time scale of the satellite constellation: for GPS $T_1^j = T_{GPS1}$ and $T_2^j = T_{GPS2}$, for GLONASS $T_1^j = T_{GL1}$ and $T_2^j = T_{GL2}$,

ξ_1^j, ξ_2^j - the total errors in the measurement of pseudoranges.

In the geocentric coordinate system, using the method of least squares, the approximate values of the absolute coordinates of the receiving antennas are calculated as $\{x_1^0, y_1^0, z_1^0\}$ and $\{x_2^0, y_2^0, z_2^0\}$, as well as discrepancies in the time scales of the receivers: for GPS - $T_{GPS_1}^0$ and $T_{GPS_2}^0$, and for GLONASS, - $T_{GL_1}^0$ and

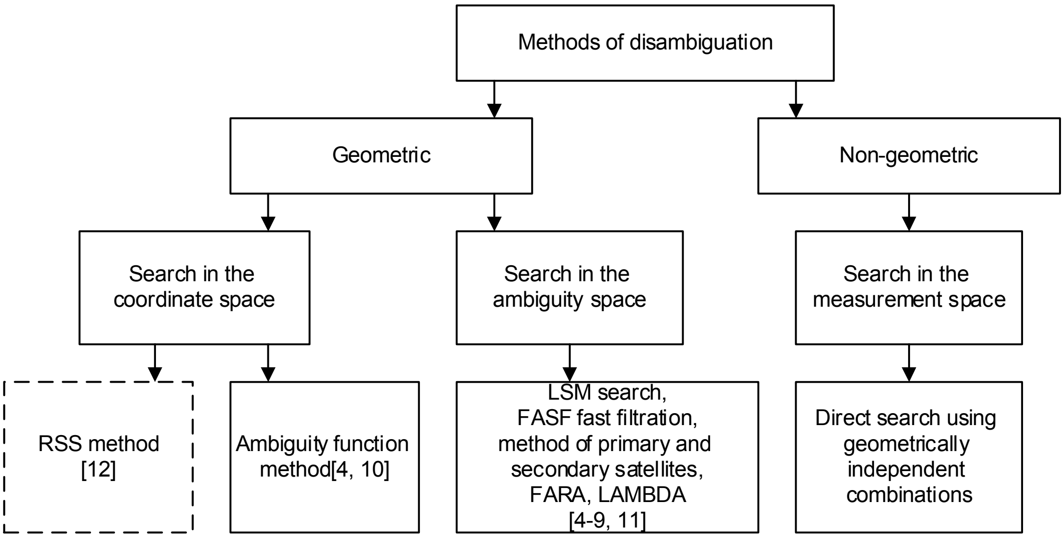

Fig. 1. Classification of methods for resolving the ambiguity of phase measurements

$T^0_{GL_2}$. The absolute coordinates found are used to refine corrections to the signal delay in the atmosphere.

When solving the navigation problem of relative positioning, the results of measurements in receivers should refer to the same time point, i.e. $t_{r1} = t_{r2} = t_r$. Then the difference of the radiation of the corresponding signals on the j-th satellite preceding this moment of time will be equal to $\Delta^j_{rad.} = \frac{1}{c}(R^j_2 - R^j_1) + \Delta t_{1.2}$,

where $\Delta t_{1.2} = T_2 - T_1$ is the divergence of the receiver time scales. In this case, in calculating $(R^j_2 - R^j_1)$, both the movement of the satellites along the orbit during the time Δt^j_{rad}, and the difference in the motion of the receivers due to the rotation of the Earth should be taken into account [13].

Expressions for the first differences of pseudo-phase measurements (in phase cycles), taking into account corrections for the delay of signals in the troposphere, ionosphere, known hardware delays, including calibration corrections for GLONASS interlater delay, etc. have the following form:

$$\Phi^j_{1.2} = \frac{R^j_2}{c} f^j_2 - \frac{R^j_1}{c} f^j_1 + f^j_0 \Delta t^j_{1.2} - \psi^j_{1.2} + N^j_{1.2} + \xi^j_{1.2},$$
$$j = 1, \ldots, n, \quad (3)$$

where R^j_1, R^j_2 are the true distances between the phase center of the transmitting antenna of the *j*-th satellite at the moment of radiation of the navigation signal and the phase centers of the first and second receiving antennas at the moments of receiving this signal (via the satellite constellation),

f^j_1 and f^j_2 are the frequencies of signals received by receivers 1 and 2 of the *j*-th satellite (taking into account the Doppler frequency shift),

f^j_0 is the nominal frequency of the signal emitted by the GPS satellites, or the frequency of the GLONASS satellite zero-letter signal,

$\psi^j_{1.2}$ is the difference of initial phases in synthesizers of reference signals of receivers in terms of carrier frequency of GPS and zero letter of GLONASS frequency (for GPS - $\psi^j_{1.2} = (\psi_{1.2})_{GPS}$, for GLONASS - $\psi^j_{1.2} = (\psi_{1.2})_{GL}$),

$N^j_{1.2}$ are unknown integers equal to the differences of the integers of the phases of the signals of the reference oscillators in the counters of the total phase measurement of receivers 1 and 2, which determine their initial state at the time of measurement,

$\xi^j_{1.2}$ is the difference in the total errors of pseudo-phase measurements in receivers due to multipath errors, noise, uncompensated delays in the atmosphere, and so on.

The divergence of the receiver time scales will be: in the GPS paths $\Delta t^j_{1.2} = T^0_{GPS_1} - T^0_{GPS_2}$, and $-\Delta t^j_{1.2} = T^0_{Гл_1} - T^0_{GL_2}$ in the GLONASS paths.

1If we assume that the origin of the required vector $\bar{L}^*$ of the baseline is at the reference point $\{x^0_1, y^0_1, z^0_1\}$, then the coordinates of its end $\{x^*_2, y^*_2, z^*_2\}$ lie in the region Q with center at the point $\{x^0_2, y^0_2, z^0_2\}$, the size

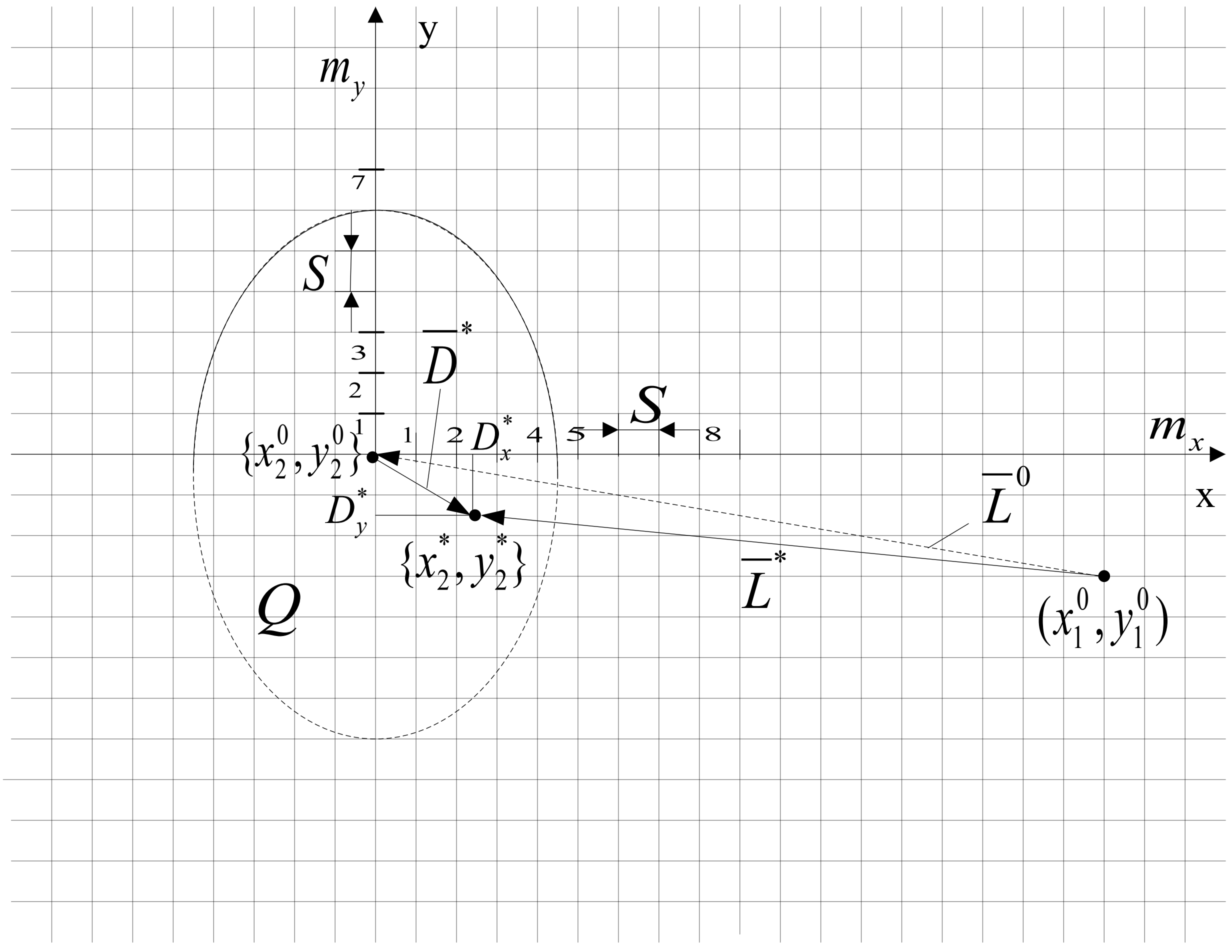

Fig. 2. Geometric relations when finding the vector of the baseline in the plane

of the domain is determined by the total errors in the calculation of the coordinates of the receiving antennas by code measurements. In this case, the end of the vector will be shifted from the coordinates of the center of the region Q by the unknowns $\{D_x^*, D_y^*, D_z^*\}$, which determine the displacement vector $\overline{D}^*$. Fig. 2 explains the geometric relationships when finding the vector of the baseline in the plane.

Displacements $\{D_x^*, D_y^*, D_z^*\}$ can be found from the first differences of pseudo-phase measurements, if we substitute in (3)

$$R_1^j = R_1^{j,0} = \sqrt{(x^j - x_1^0)^2 + (y^j - y_1^0)^2 + (z^j - z_1^0)^2}, \quad (4)$$

$$R_2^j = \sqrt{(x^j - x_2^0 - D_x)^2 + (y^j - y_2^0 - D_y)^2 + (z^j - z_2^0 - D_z)^2}, \quad (5)$$

and solve the system of nonlinear equations.

To solve the system of equations (3), the measurement times in receivers 1 and 2 must coincide. We shall calculate $R_1^{j,0}$ by the formula (4) for the coordinates of antenna 1 at the time t_{r1} (in the time scale of receiver 1) and the coordinates of the j-th satellite at the previous moments of the emission of the corresponding signals, taking into account the shift in the time scale of receiver 1 from the time scale of the satellite constellations T_{GPS1}^0 or T_{GL1}^0. We shall find the estimate $\tilde{R}_2^j$ by the formula (5) for the coordinates of the antenna 2 at the same point of time, but for the coordinates of the j-th satellite, recalculated along its orbit at time points shifted by the value $\Delta t_{1,2}^j$ from the moments, preceding t_r. In this case, the motion of each of the receivers relative to the j-th satellite as a result of the rotation of the Earth is taken into account when calculating the satellite coordinates from its ephemerides by changing from an inertial to a geodesic coordinate system [13].

Expressing the phase in meters (in wavelengths), system (3) can be represented as:

$$\widetilde{\Phi}^j_{1,2} = (\widehat{R}^j_2 - R^{j,0}_1) - g^j_{\text{GPS}}\lambda_{\text{GPS}}(M^j_{\text{GPS}} + \eta_{\text{GPS}}) - \\ - g^j_{\Gamma\text{л}}\lambda_{\Gamma\text{л}}(M^j_{\Gamma\text{л}} + \eta_{\Gamma\text{л}}) + \xi^j_{1,2}, \quad j = 1, \ldots, n, \quad (6)$$

where $\widetilde{\Phi}^j_{1.2} = \frac{c}{f^j_2}\Phi^j_2 - \frac{c}{f^j_1}\Phi^j_1$ is the difference of the pseudophases,

g^j_{GPS}, g^j_{GL} are the coefficients determining the belonging of the j-th equation to the GPS or GLONASS satellite constellation, namely

$$g^j_{GPS} = \begin{cases} 1 \; for \; j = 1, \ldots, n_{GPS} \\ 0 \; for \; j = n_{GPS} + 1, \ldots, n \end{cases}, \; g^j_{GL} = \begin{cases} 0 \; for \; j = 1, \ldots, n_{GPS} \\ 1 \; for \; j = n_{GPS} + 1, \ldots, n \end{cases},$$

M^j_{GPS}, M^j_{GL} are unknown integers,

η_{GPS} *and* η_{GL} are the unknown fractional parts of the difference of the initial phases on the carrier frequency of GPS and the zero frequency letter of GLONASS,

λ_{GPS} is the wavelength of the GPS carrier signal,

λ_{GL} is the wavelength of the zero letter carrier signal of GLONASS.

Note that the non-linearity of equation (6) with respect to the unknown displacements $\{D_x, D_y, D_z\}$ occurring in R^j_2 (5), is determined only by the first term equal to the difference in distances due to the spatial diversity of the receiver antennas. The second (for GPS) and the third (for GLONASS) terms are linear with respect to the unknowns $\{M^j \; and \; \eta\}$.

In the vicinity of the point $\{x^0_2, y^0_2, z^0_2\}$, the expressions for estimating the range (5) can be linearized:

$$\hat{R}^j_2 = R^{j\,0}_2 + A^j_x D_x + A^j_y D_y + A^j_z D_z, \quad j = 1, \ldots n, \quad (7)$$

where $R^{j\,0}_2$ is calculated by (2) with $x_2 = x^0_2$, $y_2 = y^0_2$, $z_2 = z^0_2$, and

$A^j_x = \frac{\partial R^j_2}{\partial x_2}$, $A^j_y = \frac{\partial R^j_2}{\partial y_2}$, $A^j_z = \frac{\partial R^j_2}{\partial z_2}$ are the values inverse to the direction cosines from the point $\{x^0_2, y^0_2, z^0_2\}$ to the j-th satellite.

For fixed values of the displacements $\{D_x, D_y, D_z\}$, expression (7) allows one to find estimates of the distances $\hat{R}^j_2$. Substituting $\hat{R}^j_2$ in (6), we obtain the corresponding estimates for the difference of the pseudo-phases $\hat{\Phi}^j_{1.2}$, which allow us to form residuals $(\tilde{\Phi}^j_{1.2} - \hat{\Phi}^j_{1.2})$ and write the quadratic function

$$A(D_x, D_y, D_z) = \sum_{j=1}^{n}[\tilde{\Phi}^j_{1.2} - \hat{\Phi}^j_{1.2}]^2. \quad (8)$$

Here $[x]$ means the operation of pointing off the decimal places of the x (expressed in wavelengths) of less than half the wavelength.

To explain the possibility of finding displacements $\{D_x, D_y, D_z\}$ using only fractional parts of the difference of pseudophases $\widetilde{\Phi}^j_{1.2}$, we assume that their values and parameters η_{GPS}, η_{GL} are known and equal to $\{D^*_x, D^*_y, D^*_z\}$ and η^*_{GPS}, η^*_{GL} (The required point has the coordinates $\{x^*_2, y^*_2, z^*_2\}$). For $M^j_1 = M^j_2 = 0$, substituting η^*_{GPS} and η^*_{GL} in (6) and sorting with a small step (for example, $0{,}01\lambda$) all values $\{D_x, D_y, D_z\}$ in the region Q, including $\{D^*_x, D^*_y, D^*_z\}$ (see Figure 2), we can construct the function (8). The function (8) turns out to be multimodal, and the coordinates of its main minimum will correspond to the required displacements $\{D_x, D_y, D_z\}$. Thus, the problem of determining the baseline vector from the fractional values of the first differences of pseudo-phase measurements reduces to the problem of finding the displacement coordinates $\{D_x, D_y, D_z\}$, which minimize the quadratic function (8).

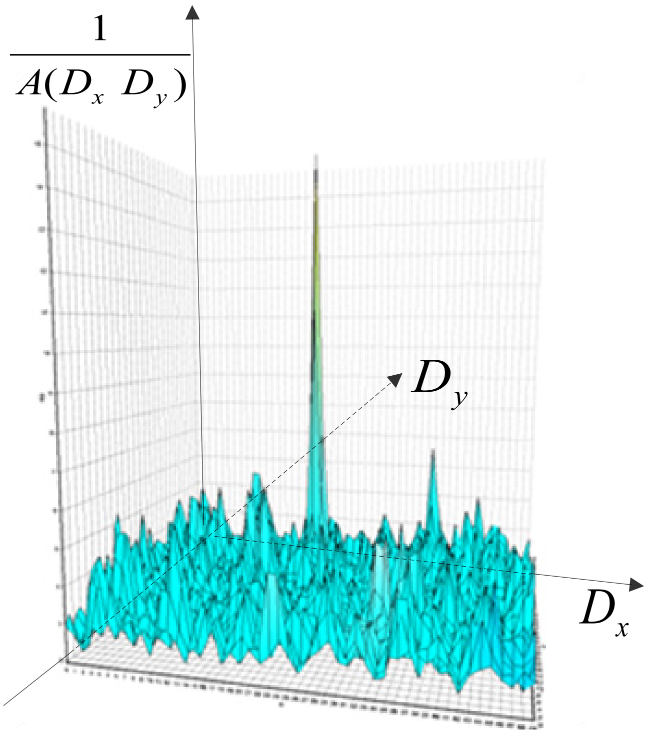

Fig. 3. A fragment of the multimodal function $\frac{1}{A(D_x, D_y)}$

In Fig. 3 for the two-dimensional case $\{x,y\}$, the function $\frac{1}{A\,(D_x,D_y)}$ is shown for clarity, the coordinates of the maxima of which coincide with the coordinates of the minima of $A\,(D_x,D_y)$. The function is constructed with the number of simultaneously visible satellites $n = 14$ and moderate values $\xi^j_{1,2}$ (not exceeding 0.1λ, that is 2 cm).

Algorithm for calculating the baseline

The patent [12] proposes a method and a device that allows one to determine the desired displacements $\{D_x, D_y, D_z\}$ from the first differences of pseudo-phase measurements of one epoch. To this end, in the region Q parallel to the coordinate axes we draw planes with step S, starting from $\{x^0_2,\ y^0_2,\ z^0_2\}$, the points of intersection of which form nodes with the coordinates $\{m_x S, m_y S, m_z S\}$, where m_x, m_y, m_z are the node numbers along the x, y, z axes (see Figure 2 for the two-dimensional case).

It will be shown below how, by analyzing the values of the function (8) in the vicinity of these nodes, one can find the coordinates of its local minima and determine the fundamental minimum. In the simplest case, when signals of wavelength λ are used and there are no measurement errors, the search step, coinciding with the grid spacing S, can be selected from the condition that spheres with radius λ and centers at the nodes fill the Q region without voids. It can be shown that in this case

$$S = \frac{\lambda}{\sqrt{3}}.$$

Below are the calculations of the probability of correct resolution of the ambiguity as a function of the step S for different values of the measurement error.

The fundamental minimum of the function (8) lies in the vicinity of one (or several) of the nodes, appearing, as a rule, displaced from it by some amount $\{D_x, D_y, D_z\}$.

For clarification, we refer to Fig. 3, which illustrates the dependence of the residuals $\tilde{\Phi}^j_{1.2} - \tilde{\Phi}^j_{1.2}(D_x)$ for several satellites on the displacement D_x, provided that along the other axes the required displacements are found and are equal to D^*_y, D^*_z. The origin ($D_x = 0$) in the figure corresponds to the point $\{x^0_2, y^0_2, z^0_2\}$, in the vicinity of which the linearization of the range estimate $\hat{R}^j_2$ (7), and the scale for both axes is chosen to be the same.

The residuals for all satellites are straight lines, the slope of which is determined by the geometric factor and can not exceed 45°. If there is no error in the measurements, all the lines intersect at the point $D_x = D^*_x$. The presence of measurement errors transforms the intersection of the residuals into a region, the dimensions of which depend on the errors in the measurement of the first differences in the pseudo-phase measurements.

If the destabilizing factors (atmosphere, calibration, reception paths, etc.) are correctly taken into account, these errors are mainly determined by the phase multipath. In practice, the coordinates of the reference point $\{x_1, y_1, z_1\}$ are chosen assuming the clear sky and the small multipath. Therefore, the multipath in the first differences is more dependent on the reception of the signal at point 2. Note that in the case of equality of the reflected signal to the direct (100% multipath), the error in determining the phase of the signal corresponds to $\pm 45°$, i.e. $\pm\frac{\pi}{4}$.

For the coordinates of the l-th node, equal to

$$D^l_x = m^l_x S,\ D^l_y = m^l_y S,\ D^l_z = m^l_z S, \tag{9}$$

the phase estimates (6) with allowance for (7) for $M^j_{GPS} = M^j_{GL} = 0$ and $\eta_{GPS} = \eta_{GL} = 0$ will be

$$(\hat{\Phi}^j_{1.2})^{l,r=0} = \hat{R}^{j,0}_2 + A^j_x\, m^l_x S + A^j_y\, m^l_y S + A^j_z\, m^l_z S - R^{j,0}_1,$$
$$j = 1, \dots, n. \tag{10}$$

The values of the deviations $\{d^{l,r=1}_x,\ d^{l,r=1}_y,\ d^{l,r=1}_z\}$ of the local minimum from the coordinates of the node (9), as well as the values $\eta^{l,r=1}_{GPS}$ and $\eta^{l,r=1}_{GL}$ can be found by solving the system of equations using the method of the least squares:

$$[(\hat{\Phi}^j_{1.2})^{l,r=1}] = [(\tilde{\Phi}^j_{1.2} - (\hat{\Phi}^j_{1.2})^{l,r=0}],\ j = 1, \dots, n. \tag{11}$$

Here

$$\begin{aligned}(\widehat{\Phi}^j_{1,2})^{l,r=1} &= \widehat{R}^{j,0}_2 + A^j_x(m^l_x S + d^{l,r=1}_x) + \\ &+ A^j_y(m^l_y S + d^{l,r=1}_y) + A^j_z(m^l_z S + d^{l,r=1}_z) - \\ &- R^{j,0}_1 - g^j_{GPS}\lambda_{GPS}\eta^{l,r=1}_{GPS} - g^j_{Гл}\lambda_{Гл}\eta^{l,r=1}_{Гл}, \\ &j = 1, \dots, n\end{aligned} \tag{12}$$

the phase estimate (6) for the coordinates of the point

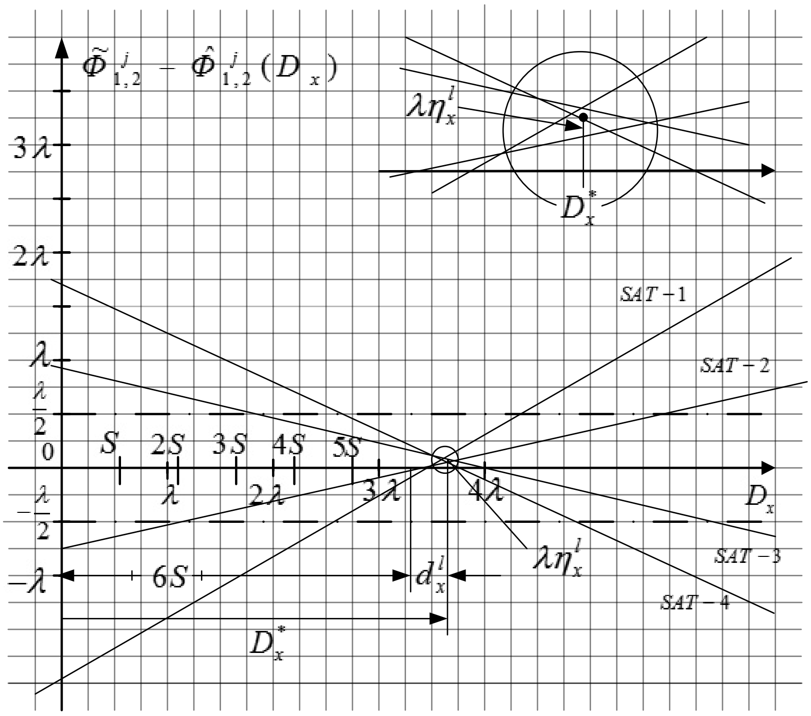

Fig. 4. Example of the dependence of the residuals on the displacement (for $D^*_x = 6S + d^l_x, D_y = D^*_y, D_z = D^*_z$)

$$D_x^{l,r=1} = m_x^l S + d_x^{l,r=1},\quad D_y^{l,r=1} = m_y^l S + d_y^{l,r=1},$$
$$D_z^{l,r=1} = m_z^l S + d_z^{l,r=1}. \tag{13}$$

Substituting the values found for $d_x^{l,r=1}, d_y^{l,r=1}, d_z^{l,r=1}, \eta_{\mathrm{GPS}}^{l,r=1}, \eta_{\mathrm{GL}}^{l,r=1}$ in (12), we obtain the estimate of the phase $(\bar{\Phi}^j_{1.2})^{l,r=1}$ on the first pass ($r = 1$) of searching for a local minimum in the vicinity of the l-th node.

To clarify the position of this minimum, we perform the second pass ($r = 2$) for the coordinates

$$D_x^{l,r=2} = m_x^l S + d_x^{l,r=1} + d_x^{l,r=2},$$
$$D_y^{l,r=2} = m_y^l S + d_y^{l,r=1} + d_y^{l,r=2},$$
$$D_z^{l,r=2} = m_z^l S + d_z^{l,r=1} + d_z^{l,r=2}, \tag{14}$$

where $d_x^{l,r=2}$, $d_y^{l,r=2}$, $d_z^{l,r=2}$ are the required corrections.

The above corrections, as well as the parameters $\eta_{\mathrm{GPS}}^{l,r=2}, \eta_{\mathrm{GL}}^{l,r=2}$ are found in the similar way as it was done on the first pass ($r = 1$), from the solution of the system of linear equations

$$[(\bar{\Phi}^j_{1.2})^{l,r=2}] = [(\tilde{\Phi}^j_{1.2} - (\bar{\Phi}^j_{1.2})^{l,r=1}],$$
$$j = 1, \ldots, n. \tag{15}$$

Let us check that the values of the corrections found are sufficiently small, for example,

$$d_x^{l,r=2} < 10^{-4}\ \text{m},\ d_y^{l,r=2} < 10^{-4}\ \text{m},$$
$$d_z^{l,r=2} < 10^{-4}\ \text{m}.$$

If all the conditions are satisfied, then we will consider the search for the coordinate shifts of the second receiving antenna at the l-th step complete, if not, then proceed to the calculation of the next ($r = 3$) correction.

(Note: As practice shows, if the number of passes in the calculation of corrections exceeds 3, then the calculation continues to be unsuitable due to the presence of one or more anomalous phase measurements. The measurements in this case should be rejected).

We denote the corrections to the shifts of the coordinates found at the l-th step as D^l_x, D^l_y, D^l_z, and the parameters $\eta^l_{\mathrm{GPS}}, \eta^l_{\mathrm{GL}}$, and introduce the residual vector as follows:

$$\nabla^{j,l} = [\tilde{\Phi}^j_{1.2} - (\bar{\Phi}^j_{1.2})^l] - \lambda_{\mathrm{GPS}}\eta^l_{\mathrm{GPS}},$$

$j = 1, \dots, n_{GPS}$ – for the GPS measurements,

$$\nabla^{j,l} = [\tilde{\Phi}^{j}_{1.2} - (\tilde{\Phi}^{j}_{1.2})^{l}] - \lambda_{GL}\eta^{l}_{GL},$$

$j = n_{GPS} + 1, \dots, n$ – for the GLONASS measurements.

The values of the residuals $\nabla^{j,l}$ depend only on the spatial separation of the receiving antennas and do not depend on the divergence of the receiver time scales, including the fractional values of the difference of the initial phases $\lambda_{GPS}\eta^{l}_{GPS}$ and $\lambda_{GL}\eta^{l}_{GL}$ (see Fig. 4).

Let us calculate the value of the quadratic function (8) at the *l*-th step

$$A^{l}(D^{l}_{x}, D^{l}_{y}, D^{l}_{z}) = \sum_{j=1}^{n} (\nabla^{j,l})^{2}$$

and compare it with the a priori given threshold a. If $\sqrt{A^{l}} \leq a$, then we assume that the fundamental minimum of the function (8) coincides with the local minimum found at the *l*-th step, and take the coordinates of the second receiving antenna found at this step of the search for the required, otherwise we continue to the $(l+1)$-th step. If for all search steps $\sqrt{A^{l}} > a$, then for the basic minimum of the function (8) we take the coordinates defined at the search step $l = l^{*}$, for which $\sqrt{A^{l}}$ is minimal.

The first case corresponds to the decision on the threshold, the second is on the absolute minimum.

We calculate the coordinates of the vector of the basis line for the coordinates of the vector of the basis line found at the l^{*}-th step of the search for coordinates of the second receiving antenna: $L_x = x^{0}_{2} + D^{l}_{x} - x^{0}_{1}$, $L_y = y^{0}_{2} + D^{l}_{y} - y^{0}_{1}$, $L_z = z^{0}_{2} + D^{l}_{z} - z^{0}_{1}$, which determine the mutual position of the objects.

If necessary, the value of M^{j}, corresponding to an integer number of cycles (wavelengths) in the pseudo-phase difference for the j-th satellite, can be determined from expression (6), substituting into it the found values $R^{j}_{2} - R^{j,0}_{1}$, and η_{GPS} or η_{GL} .

Probabilistic characteristics of the RSS method

In Fig. 5 for the total number of GPS satellites and GLONASS satellites equal to 17, the probability dependences P_{cr} of the correct resolution of the ambiguity (finding the basic minimum of the function (8)) are plotted as a function of the search step *S* for different values of the standard mean-square error $\frac{\sigma_\xi}{\lambda}\frac{\sigma_\xi}{\lambda}$ for the search area *Q* in the form of a sphere with a radius of 2 m and making a decision on the absolute minimum.

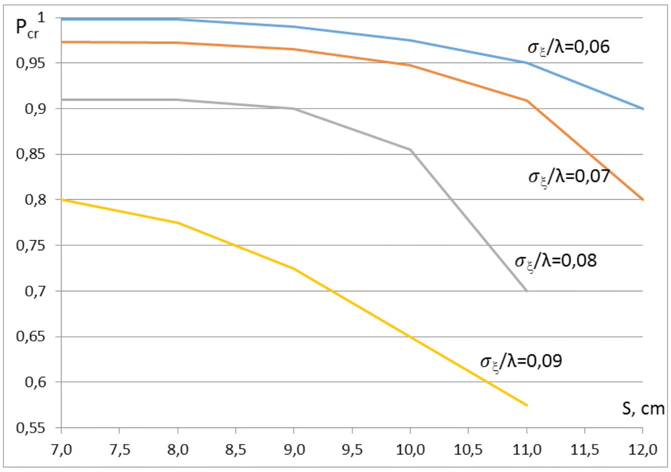

Fig. 5. The probability of correct resolution of ambiguity as a function of the search step *S* for different values of $\frac{\sigma_\xi}{\lambda}$. The search area is a sphere with a radius of 2 m

Table 1. Computing costs for the implementation of the search algorithm

1	2	3	4	5	6
S, cm	Maximum number of steps	Number of operations		Calculation time, ms 1 GHz processor	
		brute force enumeration	optimized enumeration	without using SSE	using SSE2
7	185192	123523064	55554600	111	28
8	132650	88477550	39795000	80	20
9	91124	60779708	27337200	54	14
10	68920	45969640	20676000	40	10
11	50652	33784884	15195600	32	8
12	35936	23969312	10780800	21	5.5

In Table 1, for the same conditions, the maximum number of search steps, the number of operations (addition, multiplication) for direct and optimized searches, and the calculation time on a 1 GHz processor for optimized enumeration (with and without using the SSE technology) are specified for the same conditions.

In this case, the optimization consisted in performing the full search procedure in two steps: first for measurements by GPS satellites, and then (in a reduced area) for all measurements, including GLONASS.

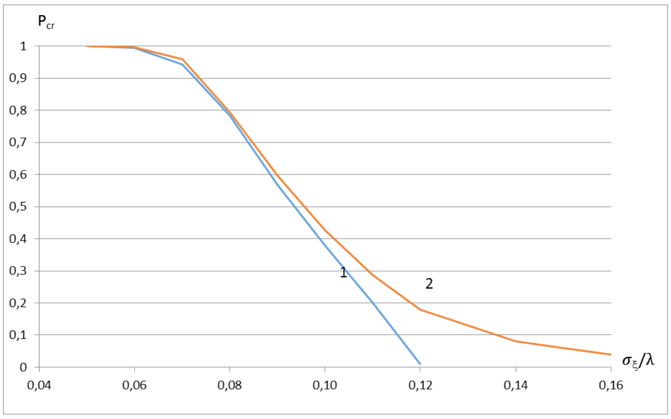

Fig. 6. Comparison of the probability of correct resolution of ambiguity when deciding on the threshold $a = 1.5$ cm (curve 1) and the absolute minimum (curve 2).

The graphs in Fig. 6 illustrate differences in the values of P_{cr} when making a decision on the threshold and on the absolute minimum, depending on $\frac{\sigma_\xi}{\lambda}$. The graphs are plotted for the search area Q in the form of a sphere with a radius of 20 cm and $S = 7$ cm. The relative small search area is characteristic for the tracking mode, when it is possible to predict the change in the relative position of objects from the results of previous measurements.

The area and strategy of the search depend on the quality of the initial measurements, the dynamics of the objects and the requirements for the results. The choice and optimization of them are beyond the scope of this article.

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